

U S A Mathematical Talent Search

PROBLEMS

Round 1 - Year 12 - Academic Year 2000-2001

1/1/12. Determine the smallest five-digit positive integer N such that $2N$ is also a five-digit integer and all ten digits from 0 to 9 are found in N and $2N$.

2/1/12. It was recently shown that $2^{2^{24}} + 1$ is not a prime number. Find the four rightmost digits of this number.

3/1/12. Determine the integers a , b , c , d , and e for which

$$(x^2 + ax + b)(x^3 + cx^2 + dx + e) = x^5 - 9x - 27.$$

4/1/12. A sequence of real numbers s_0, s_1, s_2, \dots has the property that

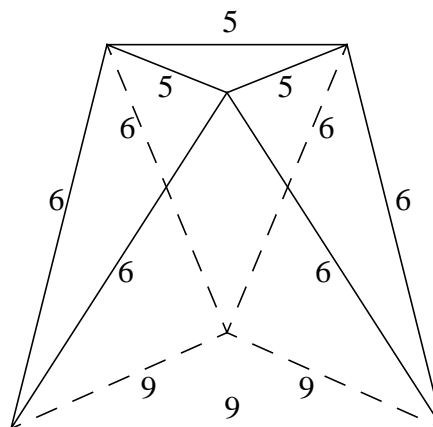
$$s_i s_j = s_{i+j} + s_{i-j} \text{ for all nonnegative integers } i \text{ and } j \text{ with } i \geq j,$$

$$s_i = s_{i+12} \text{ for all nonnegative integers } i, \text{ and}$$

$$s_0 > s_1 > s_2 > 0.$$

Find the three numbers s_0 , s_1 , and s_2 .

5/1/12. In the octahedron shown on the right, the base and top faces are equilateral triangles with sides measuring 9 and 5 units, and the lateral edges are all of length 6 units. Determine the height of the octahedron; i.e., the distance between the base and the top face.



Complete, well-written solutions to at least two of the problems above, accompanied by a completed **Cover Sheet** and a completed **Entry Form** (both available on the web site <http://www.nsa.gov/programs/mepp/usamts.html>), should be sent to the following address and **post-marked no later than September 11, 2000**. Each participant is expected to develop solutions without help from others.

USA Mathematical Talent Search
COMAP Inc., Suite 210
57 Bedford Street
Lexington, MA 02173